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INFLUENCE OF REFRACTIVE INDEX ON REFLECTANCE FROM A SEMI-INFINITE ABSORBING–SCATTERING MEDIUM WITH COLLIMATED INCIDENT RADIATION*

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Abstract—Directional and hemispherical reflectance and transmittance, which result from collimated incident radiation on a semi-infinite absorbing–scattering medium, are presented as a function of refractive index and scattering albedo. Exponential kernel substitution is used to develop an approximate but closed form solution to the governing transport equation. It is considered that scattering is isotropic and that the Fresnel relations govern the interface reflection and transmission. Exact solution, available only for normally incident radiation, reports hemispherical reflectances that are within 15 per cent of the predicted values.

NOMENCLATURE

$A(\mu)$, function defined in equation (25);
a, constant used in exponential approximation;
$B_1(\tau)$, interface function equation (11);
b, constant used in exponential approximation;
C_1, C_2, integration constants defined in equations (19) and (20);
c_o, constant defined in equation (9);
d_o, constant defined in equation (17);
$E_1(\tau)$, exponential integral of the first kind, equation (4);
F_c, collimated incident radiation;
$F(\mu_o)$, incident flux due to collimated radiation in a direction of μ_o;
$F^+(\tau)$, flux distribution in the positive direction;
$F^-(\tau)$, flux distribution in the negative direction;
$F_a^+(\tau)$, flux distribution in the positive direction, $n = 1$;
$F_a^-(\tau)$, flux distribution in the negative direction, $n = 1$;
$H(\mu)$, Chandrasekhar's H function;
$I^+(\tau, \mu, \Phi)$, intensity distribution in positive direction;
$I^-(\tau, \mu, \Phi)$, intensity distribution in negative direction;
$I_1^-(\mu_1, \Phi)$, intensity in negative direction at the vacuum side of interface;
n, refractive index of medium;
p, constant used in exponential approximation;
q, constant used in exponential approximation;
R, hemispherical reflectance;
R_a, hemispherical reflectance, $n = 1$;
R_a^*, exact hemispherical reflectance, $n = 1$;
$R(\mu_1)$, directional reflectance;
$R_a(\mu)$, directional reflectance, $n = 1$;
$R_a^*(\mu)$, exact directional reflectance, $n = 1$;
$R_d(\mu_1)$, contribution of interface to directional reflectance;
$R_s(\mu_1)$, contribution of scattering to directional reflectance;
$S(\tau)$, source function;
$S_a(\tau)$, source function, $n = 1$;
$T(\tau)$, transmittance.

Greek symbols

β, extinction coefficient;
δ, Dirac delta function;
θ, propagating angle, Fig. 1;
θ_0, incident angle, Fig. 1;
θ_1, apparent angle, Fig. 1;
μ, direction corresponding to angle θ and defined by $\mu = \cos\theta$;
$\rho(\mu)$, interface reflectance due to incident radiation from the medium in direction of μ;
$\rho_1(\mu_1)$, interface reflectance due to incident radiation from the vacuum in direction of μ_1;
σ, scattering coefficient;
τ, optical depth;
Φ, azimuthal angle;
$\psi(\tau)$, nondimensional source function;
$\psi_a(\tau)$, nondimensional source function, $n = 1$;
ω, scattering albedo.

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INTRODUCTION

WHEN THE refractive index of an absorbing and scattering medium differs from unity, interface reflections and refractions influence significantly the apparent radiative properties of the medium [1–5]. The directional and spectral nature of these properties play a major role in the field of remote sensing and radiant energy transfer; however, most of the existing analyses do not include the influence of refractive index. The present study examines that influence on directional and hemispherical reflectances and on the transmittance that result from collimated incident radiation on a semi-infinite plane medium, which absorbs and scatters radiation isotropically. Exponential kernel substitution similar to the one used by Armaly *et al.* [1, 2, 6, 7] is employed to obtain an approximate but closed form solution. The relatively simple form of the solution permits a more detailed look, over a wider range of variables than previously reported, into the roles played by scattering and refractive index.

The directional reflectance, which results from collimated incident radiation on a semi-infinite absorbing and isotropically scattering medium that has a refractive index of unity, has been studied by Chandrasekhar [8]. Giovanelli [9], through a numerical solution, examined the influence of refractive index by considering a case of normally incident radiation and provided values for hemispherical reflectance. The same condition was examined by an approximate method, which uses a two flux model and neglects directional behavior [10, 11]. For incident angles other than normal, the directional behavior of reflectance and flux distribution within a medium have not been considered in the open literature.

FORMULATION

The physical system considered consists of a semi-infinite planar medium which absorbs and scatters radiation isotropically. It is characterized by a spectral scattering albedo, ω (ratio of scatter, σ, to extinction, β, coefficient), and a refractive index, n. Monochromatic collimated radiation is incident at the interface in a direction $\mu_o = \cos\theta_o$ (Fig. 1). Emission from the medium at the incident frequency is considered negligible. The interface is assumed to be smooth and its reflection and transmission characteristics to be governed by Snell's law and the Fresnel relations [12].

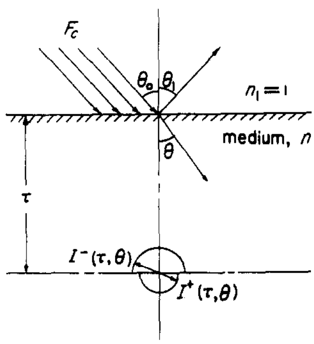

FIG. 1. Schematic of coordinate system.

The intensity distribution for the above described conditions can be stated by [2]

$$I^+(\tau,\mu,\Phi) = I^+(o,\mu,\Phi)\exp(-\tau/\mu) + \int_o^\tau S(\tau)\exp[-(\tau-t)/\mu](\mathrm{d}t/\mu) \quad (1)$$

and

$$I^-(\tau,\mu,\Phi) = \int_\tau^\infty S(\tau)\exp[-(t-\tau)/\mu](\mathrm{d}t/\mu). \quad (2)$$

The terms $I^+(\tau,\mu,\Phi)$ and $I^-(\tau,\mu,\Phi)$ denote the intensities in the plus and minus directions respectively. The term

$$\tau = \int_o^x \beta(x')\,\mathrm{d}x'$$

is the optical depth, μ equals $\cos\theta$, and $S(\tau)$ is the source function defined by

$$S(\tau) = \frac{\omega}{4\pi}\int_0^{2\pi}\int_0^1 I^+(o,\mu,\Phi)\exp(-\tau/\mu)\,\mathrm{d}\mu\,\mathrm{d}\Phi + \frac{\omega}{2}\int_0^\infty S(t)E_1(|\tau-t|)\,\mathrm{d}t. \quad (3)$$

The term $E_1(t)$ represents the exponential integral and is given by

$$E_1(t) = \int_o^1 \exp(-t/\mu)(\mathrm{d}\mu/\mu). \quad (4)$$

The intensity at the interface, $I^+(o,\mu,\Phi)$, is given by

$$I^+(o,\mu,\Phi) = n^2[1-\rho_1(\mu_1)]F_c\delta(\mu_1-\mu_o)\delta(\Phi-\Phi_o) + \rho(\mu)\int_o^\infty S(t)\exp(-t/\mu)(\mathrm{d}t/\mu). \quad (5)$$

The first term is due to the interface transmission of incident radiation, and the second is due to the interface back reflection of scattered radiation. The term $\rho_1(\mu_1)$ denotes the interface reflectance caused by radiation incident from the vacuum side in the direction μ_1, and $\rho(\mu)$ represents the interface reflectance caused by radiation incident from the medium side in the direction μ. The term $F_c\delta(\mu_1-\mu_o)\delta(\Phi-\Phi_o)$ represents the collimated incident radiation in a direction, μ_o; δ is the Dirac delta function. The directions on both sides of the interface are related by Snell's Law [12];

$$\mu_1^2 = 1-n^2(1-\mu^2). \quad (6)$$

The interface reflectances are governed by the Fresnel relations. The complex part of the index of refraction is assumed to be small in comparison to the real part. This assumption permits the interface to be treated as a pure dielectric. The reflectance, $\rho(\mu)$, equals unity for all angles larger than the critical angle, $\mu_c = \cos\theta_c$, which is given by

$$\mu_c^2 = (n^2-1)/n^2. \quad (7)$$

The transmitted incident radiation is bounded by that angle. Equations (3), (5), and (6) can be combined to yield the following integral equation for the source function:

$$S(\tau) = \omega F(\mu_o)[1-\rho_1(\mu_o)]\exp(-\tau/c_o)/4\pi c_o + \frac{\omega}{2}\int_o^\infty S(t)\left\{E_1(|\tau-t|) + \int_o^1 \rho(\mu)\exp[-(t+\tau)/\mu](\mathrm{d}\mu/\mu)\right\}\mathrm{d}t \quad (8)$$

in which

$$c_o^2 = 1-(1-\mu_o^2)/n^2 \quad (9)$$

and $F(\mu_o) = F_c\mu_o$ is the incident flux caused by collimated radiation incident in the direction of μ_o. The solution to the source function, equation (8), and its use with equations (1) and (2) lead to the complete specification of the intensity distribution within the medium.

Kernel substitution

The exact numerical solution of the governing integral equation, equation (8), is lengthy and complicated. In the present study, an exponential kernel substitution is used in an effort to obtain a closed form but approximate solution. The governing equation can be restated as

$$S(\tau) = \omega F(\mu_o)[1-\rho_1(\mu_o)]\exp(-\tau/c_o)/4\pi c_o + \frac{\omega}{2}\int_o^\infty S(t)\{E_1(|\tau-t|) + E_1(\tau+t) - B_1(\tau+t)\}\,dt, \quad (10)$$

in which

$$B_1(\tau) = \int_o^1 [1-\rho(\mu)]\exp(-\tau/\mu)(d\mu/\mu). \quad (11)$$

The function $B_1(\tau)$ appears in the governing equation, because the medium possesses a refractive index other than unity. It accounts for the back reflection to the medium at the interface. It has been established [13] that whenever the refractive index of a medium is larger than 1·2 that interface function can be approximated by an exponential term, such as

$$B_1(\tau) \simeq p\exp(-q\tau). \quad (12)$$

the constants p and q are functions of the refractive index and are tabulated by Armaly *et al.* [1, 13]. The use of these constants with the exponential approximation predicts the exact values of the function to within three per cent. When the refractive index is unity, there is no reflection or refraction at the interface, and $B_1(\tau)$ is reduced to $E_1(\tau)$. Exponential approximation to the exponential integral has been used frequently in radiative transfer studies [14] and can be represented by

$$E_1(\tau) \simeq a\exp(-b\tau). \quad (13)$$

Two choices for a and b have been used in the literature. The first, $a = b = 2$, and the second is $a = b = \sqrt{3}$. The latter has been shown to be more suitable [6] and is the one used in this study. This approximation and the one for the interface function, equation (12), are used to transform the kernel in equation (10) to a separable one.

$$\psi(\tau) = \exp(-\tau/c_o) + \frac{\omega}{2}\int_o^\infty \psi(t)\{a\exp[-b(|\tau-t|)] + a\exp[-b(\tau+t)] - p\exp[-q(\tau+t)]\}\,dt, \quad (14)$$

in which

$$\psi(\tau) = 4\pi c_o S(\tau)/\{\omega F(\mu_o)[1-\rho_1(\mu_o)]\}. \quad (15)$$

SOLUTION

The above integral equation has a separable kernel and can be transformed into an equivalent linear differential equation with constant coefficients;

$$\psi'''(\tau) + q\psi''(\tau) - d_o^2\psi'(\tau) - qd_o^2\psi(\tau) = (q-1/c_o)[(1/c_o^2)-b^2]\exp(-\tau/c_o), \quad (16)$$

in which

$$d_o^2 = b^2 - \omega ab. \quad (17)$$

The solution is given by

$$\psi(\tau) = C_1\exp(-q\tau) + C_2\exp(-d_o\tau) + (1-b^2c_o^2)\exp(-\tau/c_o)/(1-d_o^2c_o^2). \quad (18)$$

The condition that the source function is finite has been used to eliminate one constant. The remaining two constants can be evaluated by back substitution of the solution in the governing integral equation and by equating terms of equal power. The resulting expressions for these constants are:

$$C_1 = -\{\omega abc_o/[(1-c_o^2d_o^2)d_o] + c_o(1-b_o^2c_o^2)(d_o+q)/[(1+c_oq)(1-c_o^2d_o^2)]\}/\{\omega abq/[d_o(q^2-b^2)] + [2(d_o+q)/p\omega] \times [1+0{\cdot}5\omega\{2ab/(q^2-b^2)+0{\cdot}5p/q\}]\} \quad (19)$$

and

$$C_2 = (0{\cdot}5\omega ab/d_o)\{2qC_1/(q^2-b^2) + 2c_o/(1-c_o^2d_o^2)\}. \quad (20)$$

The above relations for C_1 and C_2 are not suitable when ω equals 1. For that case, the following relations should be used:

$$C_1 = c_o(b^2-q^2)/q \quad (21)$$

and

$$C_2 = -(2q/p)\{C_1[1+ab/(q^2-b^2)+p/4q] + c_op(1-b^2c_o^2)/[2(1+c_oq)]\}. \quad (22)$$

Apparent properties

The solution for the source function can now be used to evaluate the intensity distribution and to examine the influence of refractive index and scattering on the apparent radiative properties. The intensity at the vacuum side of the interface, $I_1^-(\mu_1,\Phi)$, is due to direct interface reflectance of incident radiation and to interface transmittance or scattered radiation. It is given by

$$I_1^-(\mu_1,\Phi) = \rho_1(\mu_1)F_c\delta(\mu_1-\mu_o)\delta(\Phi-\Phi_o) + \{[1-\rho(\mu)]/\mu n^2\}\int_o^\infty S(t)\exp(-t/\mu)\,dt. \quad (23)$$

Equation (18) when used with equation (23) yields the following:

$$I_1^-(\mu_1,\Phi) = \rho_1(\mu_1)F_c\delta(\mu_1-\mu_o)\delta(\Phi-\Phi_o) + \omega F(\mu_o)[1-\rho_1(\mu_o)][1-\rho(\mu)]A(\mu)/(4\pi c_on^2), \quad (24)$$

in which

$$A(\mu) = C_1/(q\mu+1) + C_2/(\mu d_o+1) + (1-b^2c_o^2)c_o/[(1-c_o^2d_o^2)(\mu+c_o)]. \quad (25)$$

The intensity, $I^+(o,\mu,\Phi)$, at the interface within the medium is due to interface transmittance of collimated incident radiation plus the internally reflected scattered radiation. It can be deduced from equation (5) as

$$I^+(o,\mu,\Phi) = n^2F_c[1-\rho_1(\mu_1)]\delta(\mu_1-\mu_o)\delta(\Phi-\Phi_o) + \omega F(\mu_o)\rho(\mu)[1-\rho_1(\mu_o)]A(\mu)/4\pi c_o. \quad (26)$$

Directional reflectance, $R(\mu_1)$, is defined as the ratio of apparent intensity, equation (24), to the incident collimated radiation, F_c. It can be viewed as the sum of two components

$$R(\mu_1) = R_d(\mu_1) + R_s(\mu_1). \quad (27)$$

The first is due to direct surface reflectance and is given by

$$R_d(\mu_1) = \rho_1(\mu_1)\delta(\mu_1 - \mu_o)\delta(\Phi - \Phi_o). \quad (28)$$

The second is due to interface transmittance of scattered radiation and is given by

$$R_s(\mu_1) = \omega\mu_o[1-\rho_1(\mu_o)][1-\rho(\mu)]A(\mu)/(4\pi c_o n^2). \quad (29)$$

Hemispherical reflectance, the ratio of reflected to incident flux, can be evaluated by using the following definition:

$$R = \int_o^{2\pi}\int_o^1 R(\mu_1)\mu_1\,d\mu_1\,d\Phi/\mu_o. \quad (30)$$

The intensity distribution within the medium, $I^+(\tau,\mu,\Phi)$ and $I^-(\tau,\mu,\Phi)$, as expressed in equations (1) and (2), respectively, can be used with the solution for the source function, equation (18), to evaluate the flux distribution and the transmittance as defined by:

$$F^+(\tau) = \int_o^{2\pi}\int_o^1 I^+(\tau,\mu,\Phi)\mu\,d\mu\,d\Phi \quad (31)$$

$$F^-(\tau) = \int_o^{2\pi}\int_o^1 I^-(\tau,\mu,\Phi)\mu\,d\mu\,d\Phi \quad (32)$$

and

$$T(\tau) = [F^+(\tau) - F^-(\tau)]/F(\mu_o). \quad (33)$$

MEDIUM WITH A REFRACTIVE INDEX OF UNITY

When the refractive index of a medium becomes equal to that of the surroundings, reflection and refraction at the interface do not take place. The interface function, $B_1(\tau)$, is reduced to the exponential integral $E_1(\tau)$. Some of the apparent properties of such a medium have been investigated in the literature [8, 9] through the exact solution of the governing equations. The exponential kernel approximation is applied to this problem in order to evaluate its applicability by comparing it with exact results and to justify its application to cases in which the refractive index is other than unity, and exact results are not available.

The integral equation governing the source function for an interface refractive index equal to unity can be deduced from equation (10) and is given by

$$\psi_a(\tau) = \exp(-\tau/\mu_o) + \frac{\omega}{2}\int_o^\infty \psi_a(t)E_1(|\tau - t|)\,dt, \quad (34)$$

in which

$$\psi_a(\tau) = 4\pi S_a(\tau)/(\omega F_c). \quad (35)$$

The subscript, a, identifies quantities associated with the case in which the refractive index is unity. By making use of the exponential approximation for $E_1(\tau)$, equation (13), the integral equation can be transformed to one with a separable kernel. The solution is given by

$$\psi_a(\tau) = \mu_o(1+\mu_o b)(d_o - b)\exp(-d_o\tau)/(\mu_o^2 d_o^2 - 1) - (1-\mu_o^2 b^2)\exp(-\tau/\mu_o)/(\mu_o^2 d_o^2 - 1). \quad (36)$$

The apparent properties can now be evaluated by using definitions stated in the previous section. The directional reflectance is given by

$$R_a(\mu) = [\omega/(4\pi)]\{\mu_o(1+\mu_o b)(d_o - b)/[(\mu_o^2 d_o^2 - 1)(\mu d_o + 1)] - \mu_o(1-\mu_o^2 b^2)/[(\mu_o^2 d_o^2 - 1)(\mu + \mu_o)]\}, \quad (37)$$

and the hemispherical reflectance becomes

$$R_a = [0.5\omega(1+\mu_o b)(d_o - b)/(\mu_o^2 d_o^2 - 1)] \times [(1/d_o) - \ln(1+d_o)/d_o^2] \times [-0.5\omega(1-\mu_o^2 b^2)/(\mu_o^2 d_o^2 - 1)] \times [1 - \mu_o\ln(\mu_o + 1) + \mu_o\ln\mu_o]. \quad (38)$$

The special case of $\omega = 1$ yields

$$R_a = 0.25b(1+\mu_o b) + 0.5(1-\mu_o^2 b^2) \times [1 - \mu_o\ln(\mu_o + 1) + \mu_o\ln\mu_o]. \quad (39)$$

The flux distribution within the medium can be expressed by

$$F_a^+(\tau) = F(\mu_o)\exp(-\tau/\mu_o) + [0.5\omega F(\mu_o)/(\mu_o^2 d_o^2 - 1)] \times \int_o^1 \{(1+\mu_o b)(d_o - b)[\exp(-\tau/\mu) - \exp(-d_o\tau)]/(\mu d_o - 1) - (1-\mu_o^2 b^2) \times [\exp(-\tau/\mu_o) - \exp(-\tau/\mu)]/(\mu - \mu_o)\}\mu\,d\mu \quad (40)$$

and

$$F_a^-(\tau) = 0.5\omega F(\mu_o)\int_o^1 \{(1+\mu_o b)(d_o - b)\exp(-d_o\tau)/[(\mu_o^2 d_o^2 - 1)(\mu d_o + 1)] - (1-\mu_o^2 b^2) \times \exp(-\tau/\mu_o)/[(\mu_o^2 d_o^2 - 1)(\mu_o + \mu)]\}\mu\,d\mu. \quad (41)$$

Giovanelli [9] treated the case in which the refractive index is unity and developed exact expressions for the above properties given by

$$R_a^*(\mu) = \omega\mu_o H(\mu)H(\mu_o)/[4\pi(\mu + \mu_o)] \quad (42)$$

and

$$R_a^* = 1 - H(\mu_o)\sqrt{(1-\omega)}. \quad (43)$$

The term $H(\mu)$ is Chandresekhar's H function, and the asterisks appended to the properties identify them as exact. These two results, exact and approximate, are compared in the next section.

RESULTS AND DISCUSSION

The contribution of scattering to the directional reflectance, $R_s(\mu_1)$, which increases with the scattering albedo, is presented in Figs. 2–4 for various optical conditions. The figures exhibit clearly the dependence of this property on the refractive index, the scattering albedo, and both the apparent and incident angles. It decreases as the refractive index increases because the back reflectance increases. This influence, however, decreases as the scattering albedo approaches unity. As the incident angle increases, the interface transmittance decreases and causes the scattering contribution to decrease. The direct surface reflectance, $R_d(\mu_1)$, is governed by the Fresnel relations and is not shown graphically. Its magnitude increases with the refractive index and the incident angle.

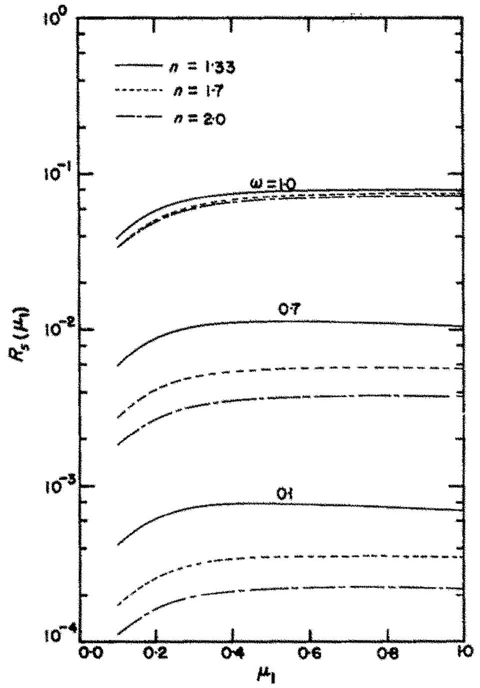

FIG. 2. Contribution of scattering to directional reflectance, $\mu_o = 0{\cdot}3$.

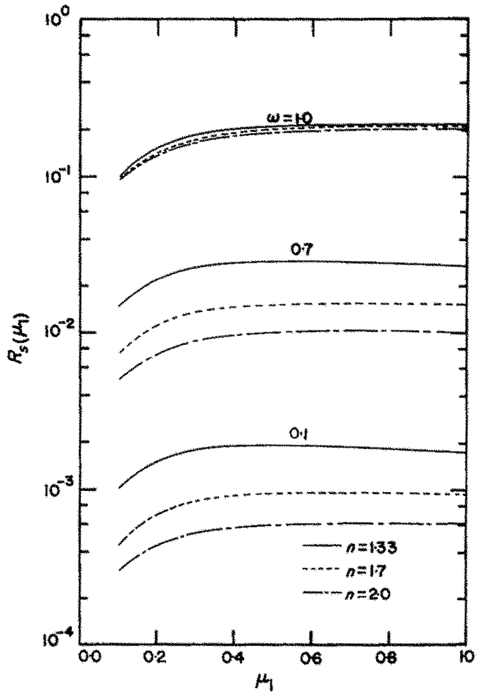

FIG. 4. Contribution of scattering to directional reflectance, $\mu_o = 1{\cdot}0$.

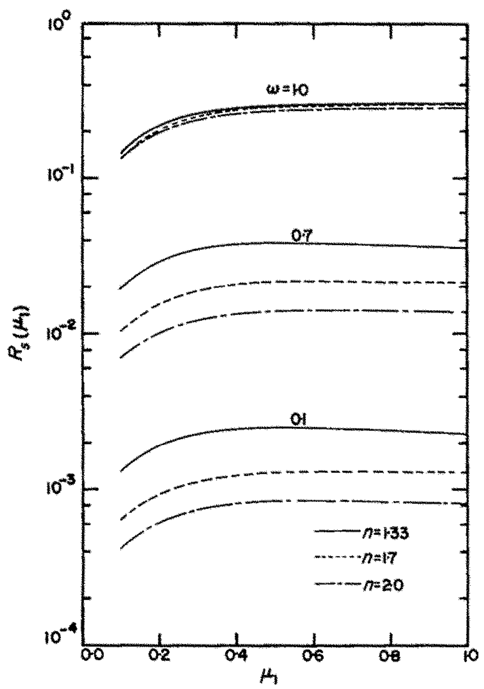

FIG. 3. Contribution of scattering to directional reflectance, $\mu_o = 0{\cdot}7$.

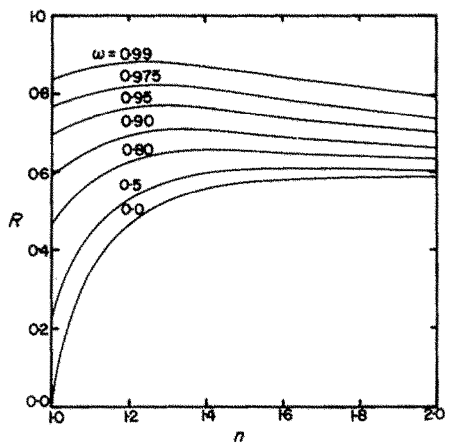

FIG. 5. Hemispherical reflectance, $\mu_o = 0{\cdot}1$.

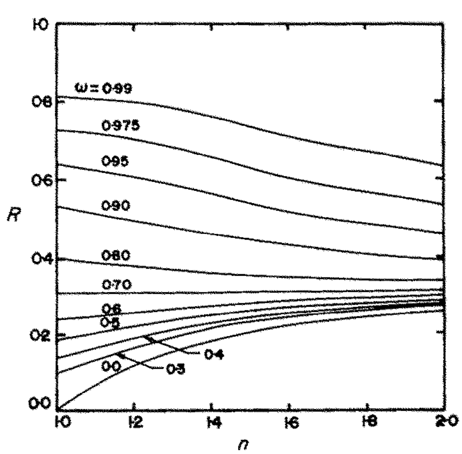

FIG. 6. Hemispherical reflectance, $\mu_o = 0{\cdot}3$.

The hemispherical reflectance is depicted in Figs. 5–7 and appears to increase with the scattering albedo and incident angle. The combined influence of the two directional reflectance components (direct surface reflectance, $R_d(\mu_1)$, and scattered contribution, $R_s(\mu_1)$) governs its behavior. When the scattering albedo is small, the hemispherical reflectance is influenced

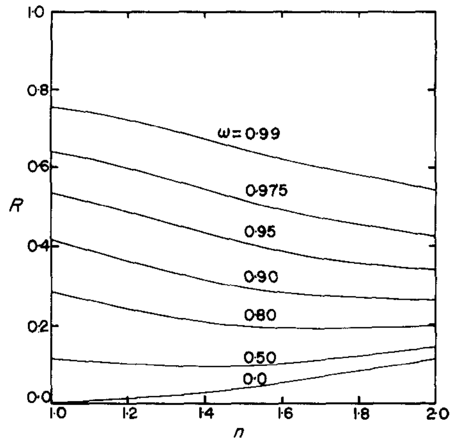

FIG. 7. Hemispherical reflectance, $\mu_o = 1{\cdot}0$.

strongly by the behavior of the direct interface reflectance, i.e. it increases with the refractive index and the incident angle. For large scattering albedo, the scattered contribution can dominate the behavior of the hemispherical reflectance, i.e. it decreases as the refractive index and incident angle increase. These two opposing trends cause, in most cases, the hemispherical reflectance to increase with the refractive index for small scattering albedo and reverse that behavior as the scattering albedo increases.

The transmittance, which is produced by incident radiation that is normal to the interface of a medium possessing a refractive index of 1·33 and 2·0, is presented in Fig. 8. For other incident angles, the transmittance would be smaller because the surface reflectance would be higher; however, the trend would be similar to the normal incident case. At low scattering albedo, the negative flux, $F^-(\tau)$, is negligible relative to the positive flux, $F^+(\tau)$, and the latter dictates the behavior of the transmittance, equation (33). The magnitude of the negative flux decreases as the refractive index increases because the surface reflectance is higher, and it decays rapidly with optical depth in a manner similar to an exponentially decaying term. As the scattering albedo increases, the negative flux increases and becomes equivalent to the positive flux at $\omega = 1$ (conservative case). This behavior reduces the magnitude of the transmittance at the interface and decreases the rate of its decay with optical depth. An increase in refractive index increases the back surface reflectance, which causes the transmittance to increase for high scattering albedo.

A comparison with the exact solution is presented in Fig. 9 for the case in which the refractive index is unity. Excellent agreement exists when the scattering albedo is small, and the error increases slightly as that

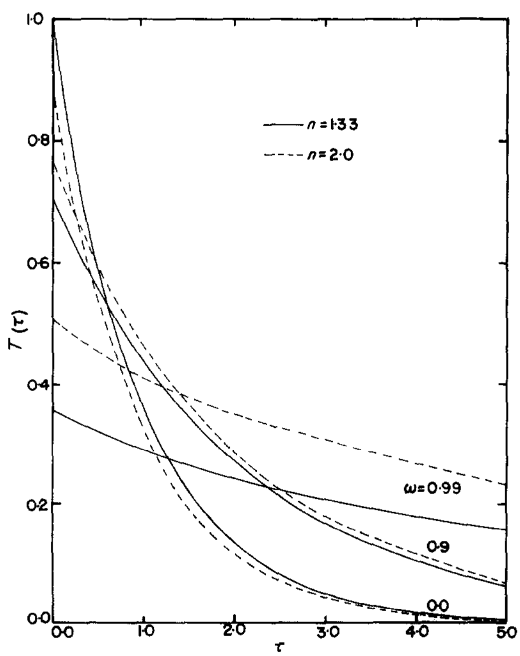

FIG. 8. Apparent transmittance, $\mu_o = 1{\cdot}0$.

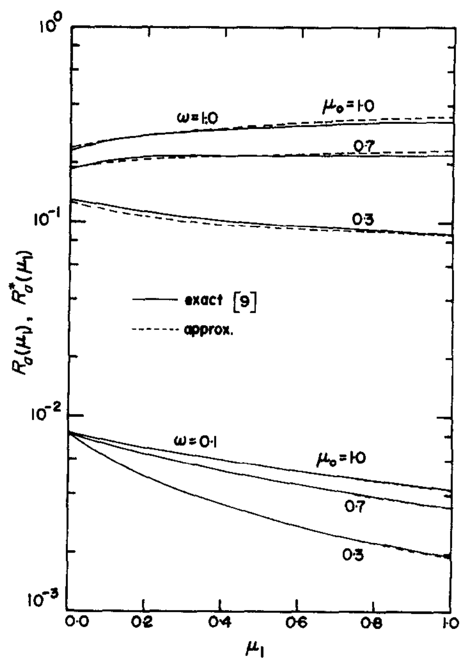

FIG. 9. Comparison between exact and approximate directional reflectance, $n = 1$.

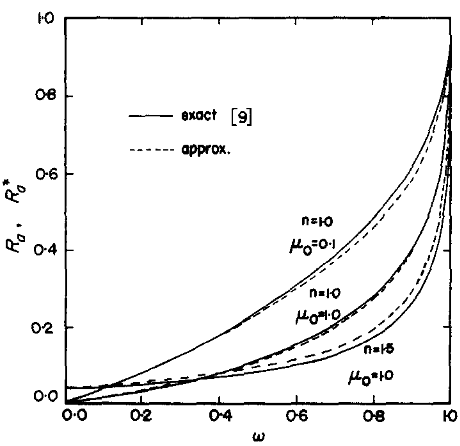

FIG. 10. Comparison between exact and approximate hemispherical reflectance.

parameter increases. When the refractive index is other than unity, exact results are available only for the hemispherical reflectance resulting from normal incident radiation. A comparison with these results is shown in Fig. 10. The predicted values differ by less than 15 per cent from the exact values.

CONCLUSIONS

The influence of the refractive index and scattering albedo on apparent directional and hemispherical reflectance and transmittance resulting from collimated incident radiation has been demonstrated. The simple, closed form, approximate solution makes it feasible to evaluate total properties and to perform radiative interchange calculations when integrations over directions and frequencies are required. Directional properties and flux distribution within a medium can be readily evaluated from the approximate solution. The contribution of scattering to the directional reflectance increases as the scattering albedo increases and decreases as both the incident angle and refractive index increase. The hemispherical reflectance increases as both the scattering albedo and incident angle increase. An increase in the refractive index will in most cases increase the hemispherical reflectance at small scattering albedo, and that trend reverses as the scattering albedo increases. The transmittance decreases as the refractive index increases at low scattering albedo, and this trend is reversed at high scattering albedo. In addition, it decreases as scattering albedo increases for small optical depth, and it reverses that trend at large optical depth. The approximate method appears to predict apparent properties to within 15 per cent of those predicted by the exact solution.

REFERENCES

1. B. F. Armaly, T. T. Lam and A. L. Crosbie, Emittance of semi-infinite absorbing and isotropically scattering medium with refractive index greater than unity, *AIAA Jl* **11**(11), 1498–1502 (November 1973).
2. B. F. Armaly and T. T. Lam, Reflectance from semi-infinite absorbing-scattering medium with diffuse incident radiation, in *AIAA Progress in Astronautics and Aeronautics: Thermal Control and Radiation* Vol. 35, edited by R. G. Hering. MIT Press, Cambridge, Mass. (1974).
3. W. D. Turner and T. J. Love, Directional emittance from a two-dimensional ceramic coating, *AIAA Jl* **9**(9), 1849–1853 (September 1971).
4. W. D. Turner and T. J. Love, Directional emittance of a one-dimensional absorbing–scattering slab with reflecting boundaries, AIAA Paper No. 72–309, Presented at the 7th AIAA Thermophysics Conference, San Antonio, Texas (February 10–12, 1972).
5. R. Viskanta and J. S. Toor, Radiant energy transfer in waters, *Wat. Resourc. Res. Jl* **8**(3), 595–608 (June 1972).
6. B. F. Armaly and T. T. Lam, A note on the exponential kernel approximation, *Jl Quantve Spectros. & Radiat. Transf.* **14**(7), 651–656 (July 1974).
7. T. T. Lam, MS Thesis, University of Missouri-Rolla (1973).
8. S. Chandrasekhar, *Radiative Transfer*. Dover, New York (1960).
9. R. G. Giovanelli, Reflection by semi-infinite diffusers, *Opt. Ac.* **2**(4), 153–162 (December 1955).
10. G. Kortum, *Reflectance Spectroscopy*. Springer, Berlin (1969).
11. W. R. Belvin and W. J. Brown, Total reflectances of opaque diffusers, *J. Opt. Soc. Am.* **52**(11), 1250–1255 (November 1962).
12. M. Born and E. Wolf, *Optics*. Addison-Wesley, New York (1950).
13. B. F. Armaly, T. T. Lam and A. L. Crosbie, A note on the interface functions, *Jl Quantve Spectsos. & Radiat. Transf.* **13**(8), 813–819 (August 1973).
14. E. M. Sparrow and R. D. Cess, *Radiative Heat Transfer*. Brooks/Cole, Monterey, Ca. (1966).

INFLUENCE DE L'INDICE DE REFRACTION SUR LE FACTEUR DE REFLEXION D'UN MILIEU DIFFUSIF ABSORBANT SEMI-INFINI AVEC RAYONNEMENT INCIDENT COLLIMATE

Résumé—Le facteur de réflexion et la transmittance directionnels et hémisphériques, qui résultent d'un rayonnement collimaté incident sur un milieu semi-infini diffusif absorbant, sont présentés comme des fonctions de l'indice de réfraction et de l'albedo de diffusion. Une substitution à noyau exponentiel est utilisée afin de développer une solution approchée mais qui permet la fermeture de l'équation fondamentale de transport. On admet que la diffusion est isotrope et que les relations de Fresnel gouvernent la réflexion et la transmission sur l'interface. Une solution exacte, utilisable seulement pour un rayonnement incident normal, fournit des facteurs de réflexion hémisphériques qui approchent les valeurs prévisionnelles à 15 pour cent près.

DER EINFLUSS DES BRECHUNGSINDEX AUF DAS REFLEXIONSVERMÖGEN EINES HALBUNENDLICHEN, ABSORBIERENDEN, STREUENDEN MEDIUMS BEI GEBÜNDELT AUFTREFFENDER STRAHLUNG

Zusammenfassung—Die von einer gebündelt auf ein halbunendliches, absorbierendes, streuendes Medium auftreffenden Strahlung herrührende Reflexion und Transmission werden als Funktion des Brechungsindex und der Albedo dargestellt.

Eine exponentielle Kernsubstitution wird angewendet, um eine geschlossene Näherungslösung der maßgebenden Transportgleichung zu entwickeln. Es wird berücksichtigt, daß die Streuung isotrop ist und die Fresnel-Gleichungen die Zwischenschichtreflexion und die Transmission bestimmen. Eine exakte Lösung, die nur für Normalstrahlung anwendbar ist, wird auf die Reflexion in den Halbraum übertragen und liefert auf 15 Prozent genaue Ergebnisse.

ВЛИЯНИЕ ПОКАЗАТЕЛЯ ПРЕЛОМЛЕНИЯ НА ОТРАЖАТЕЛЬНУЮ СПОСОБНОСТЬ ПОЛУБЕСКОНЕЧНОЙ ПОГЛОЩАЮЩЕЙ И РАССЕИВАЮЩЕЙ СРЕДЫ С КОЛЛИМИРОВАННЫМ ПАДАЮЩИМ ИЗЛУЧЕНИЕМ

Аннотация — Направленные и полусферические отражения и пропускательная способность при коллимированном излучении, падающем на полупрозрачную поглощающую рассеивающую среду, представлены в зависимости от показателя преломления и альбедо рассеивания. Использована подстановка экспоненциального ядра для получения приближенного решения уравнения переноса в замкнутой форме. Считается, что рассеивание является изотропным, а зависимость Френеля описывает отражение и пропускание. Точное решение, имеющееся только для перпендикулярно падающего излучения, дает коэффициенты полусферического отражения, которые соответствуют расчетным значениям в пределах 15%.